**Formula Sheet**

**Chapter 1**

**Definition 1.1 Mean**

The mean of a sample of n measured responses y1, y2,..., yn is given by

The corresponding population mean is denoted µ

**Definition 1.2 Variance**

The variance of a sample of measurements y1, y2,..., yn is the sum of the square of the differences between the measurements and their mean, divided by n − 1. Symbolically, the sample variance is

The corresponding population variance is denoted by the symbol σ2

**Definition 1.3 Standard Deviation**

The standard deviation of a sample of measurements is the positive square root

of the variance; that is,

The corresponding population standard deviation is denoted by σ = √ σ2.

**Chapter 2**

**Definition 2.1 Experiment**

An experiment is the process by which an observation is made.

**Definition 2.2 Simple Event**

A simple event is an event that cannot be decomposed. Each simple event corresponds to one and only one sample point. The letter E with a subscript will be used to denote a simple event or the corresponding sample point.

**Definition 2.3 Sample Space**

The sample space associated with an experiment is the set consisting of all possible sample points. A sample space will be denoted by S.

**Definition 2.4 Discrete Sample Space**

A discrete sample space is one that contains either a finite or a countable number of distinct sample points.

**Definition 2.5 Event**

An event in a discrete sample space S is a collection of sample points—that is, any subset of S.

**Definition 2.6 Probability**

Suppose S is a sample space associated with an experiment. To every event A in S (A is a subset of S), we assign a number, P(A), called the probability of A, so that the following axioms hold:

Axiom 1:

Axiom 2:

Axiom 3: If A1, A2, A3, . . . form a sequence of pairwise mutually exclusive events in S (that is, if ) then

**Sample-point Method**

The sample-point method is outlined in Section 2.4. The following steps are used to find the probability of an event:

1. Define the experiment and clearly determine how to describe one simple event.

2. List the simple events associated with the experiment and test each to make certain that it cannot be decomposed. This defines the sample space S.

3. Assign reasonable probabilities to the sample points in S, making certain that P(Ei) ≥ 0 and P(Ei) = 1.

4. Define the event of interest, A, as a specific collection of sample points. (A sample point is in A if A occurs when the sample point occurs. Test all sample points in S to identify those in A.)

5. Find P(A) by summing the probabilities of the sample points in A.

**Theorem 2.1 mn = m x n**

With m elements a1, a2,..., am and n elements b1, b2,..., bn, it is possible to form mn = m × n pairs containing one element from each group.

**Proof** Verification of the theorem can be seen by observing the rectangular table in Figure 2.9. There is one square in the table for each ai, bj pair and hence a total of m × n squares.

**Definition 2.7 Permutation**

An ordered arrangement ofr distinct objects is called a permutation. The number of ways of ordering n distinct objects taken r at a time will be designated by the symbol Pn r .

**Definition 2.8 Combinations**

The number of combinations of *n* objects taken *r* at a time is the number of subsets, each of size *r*, that can be formed from the *n* objects. This number will be denoted by or ()

**Definition 2.9 Conditional Probability of an Event**

The conditional probability of an event A, given that an event B has occurred, is equal to

provided P(B) > 0. [The symbol P(A|B) is read “probability of A given B.”]

**Definition 2.10 Independent Cases**

Two events A and B are said to be independent if any one of the following holds:

Otherwise, the events are said to be dependent.

**Theorem 2.5 The Multiplicative Law of Probability**

The Multiplicative Law of Probability The probability of the intersection of two events A and B is

If A and B are independent, then

**Proof** The multiplicative law follows directly from Definition 2.9, the definition of conditional probability

**Theorem 2.6 The Additive Law of Probability**

The Additive Law of Probability The probability of the union of two events A and B is

If A and B are mutually exclusive events, P(A ∩ B) = 0 and

**Proof** The proof of the additive law can be followed by inspecting the Venn diagram in Figure 2.10. Notice that A ∪ B = A ∪ (A ∩ B), where A and (A ∩ B) are mutually exclusive events. Further, B = (A ∩ B)∪(A ∩ B), where (A ∩ B) and (A ∩ B) are mutually exclusive events. Then, by Axiom 3, P(A ∪ B) = P(A) + P(A ∩ B) and P(B) = P(A ∩ B) + P(A ∩ B). The equality given on the right implies that P(A ∩ B) = P(B) − P(A ∩ B). Substituting this expression for P(A ∩ B) into the expression for P(A ∪ B) given in the left-hand equation of the preceding pair, we obtain the desired result:

**Theorem 2.7 Mutual Exclusive Events**

If A is an event, then

**Proof** Observe that S = A ∪ A. Because A and A are mutually exclusive events, it follows that P(S) = P(A) + P(A). Therefore, P(A) + P(A) = 1 and the result follows.

**Definition 2.11 Partition**

For some positive integer k, let the sets B1, B2,..., Bk be such that

1. S = B1 ∪ B2 ∪···∪ Bk .

2. Bi ∩ Bj = ∅, for i = j.

Then the collection of sets {B1, B2,..., Bk } is said to be a partition of S.

**Theorem 2.9 Bayes’ Rule**

Bayes’ Rule Assume that {B1, B2,..., Bk } is a partition of S (see Definition 2.11) such that P(Bi) > 0, for i = 1, 2,..., k. Then

**Definition 2.12 Random Variable**

A random variable is a real-valued function for which the domain is a sample space

**Definition 2.13 Random Sample**

Let N and n represent the numbers of elements in the population and sample, respectively. If the sampling is conducted in such a way that each of the N n samples has an equal probability of being selected, the sampling is said to be random, and the result is said to be a random sample.

**Chapter 3**

**Definition 3.1 Discrete Values**

A random variable Y is said to be discrete if it can assume only a finite or countably infinite1 number of distinct values.

**Definition 3.2 Sum of the Probabilities of all Sample Points**

The probability that Y takes on the value y, P(Y = y), is defined as the sum of the probabilities of all sample points in S that are assigned the value y. We will sometimes denote P(Y = y) by p(y).

**Definition 3.3 Probability Distribution**

The probability distribution for a discrete variable Y can be represented by a formula, a table, or a graph that provides p(y) = P(Y = y) for all y.

**Definition 3.4 Expected Value**

Let Y be a discrete random variable with the probability function p(y). Then the expected value of Y , E(Y ), is defined to be2

**Definition 3.5 Random Variable w/Mean**

If Y is a random variable with mean E(Y ) = µ, the variance of a random variable Y is defined to be the expected value of (Y − µ)2. That is,

The standard deviation of Y is the positive square root of V(Y ).

**Definition 3.6 Binomial Experiment**

A binomial experiment possesses the following properties:

1. The experiment consists of a fixed number, n, of identical trials.

2. Each trial results in one of two outcomes: success, S, or failure, F.

3. The probability of success on a single trial is equal to some value p and remains the same from trial to trial. The probability of a failure is equal to q = (1 − p).

4. The trials are independent.

5. The random variable of interest is Y , the number of successes observed during the n trials.

**Definition 3.7 Binomial Distribution**

A random variable Y is said to have a binomial distribution based on n trials with success probability p if and only if

, and

**Definition 3.8 Geometric Probability**

A random variable Y is said to have a geometric probability distribution if and only if